THE INTERACTING BOSON–FERMION MODEL

F. IACHELLO

Center for Theoretical Physics
Yale University

P. VAN ISACKER

SERC Daresbury Laboratory

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THE INTERACTING BOSON–FERMION MODEL-1

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1.1 Introduction

In many cases in physics, one has to deal simultaneously with collective and single-particle excitations of the system. The collective excitations are usually bosonic in nature while the single-particle excitations are often fermionic. One is therefore led to consider a system which includes bosons and fermions. In this book we discuss applications of a general algebraic theory of mixed Bose–Fermi systems to atomic nuclei. The collective degrees of freedom here can be described in terms of a system of interacting bosons as discussed in a previous book (Iachello and Arima, 1987), henceforth referred to as Volume 1. The single-particle degrees of freedom represent the motion of individual nucleons in the average nuclear field. They are described in terms of a system of interacting fermions. The coupling of fermions and bosons leads to the interacting boson–fermion model which has been used extensively in recent years to discuss the properties of nuclei with an odd number of nucleons.

The interacting boson–fermion model was introduced by Arima and one of us in 1975 (Arima and Iachello, 1975). It was subsequently expanded by Iachello and Scholten (1979) and cast into a form more readily amenable to calculations. As in the corresponding case of even-mass systems, the algebra of creation and annihilation operators can be realized in several ways. One of these is the Holstein–Primakoff realization which leads to a slightly different version of the interacting boson–fermion model called the truncated quadrupole phonon–fermion model (Paar, 1980; Paar and Brant, 1981), based on the boson realization introduced by Janssen, Jolos and Dõnau in 1974 and discussed in Sect. 1.4.6 of

Volume 1. In this book we discuss only the algebraic and geometric properties of the interacting boson–fermion model. The microscopic origin and justification will be dealt with in a subsequent book. As in the case of even-mass systems, there are several versions of the model which differ in their treatment of the proton and neutron degrees of freedom. In the first version, called the interacting boson–fermion model-1 (IBFM-1) and discussed in Part I of this book, no distinction is made between protons and neutrons. In the other versions of the model they are treated explicitly. The interacting boson–fermion model-2 (IBFM-2) applies to nuclei where protons and neutrons occupy different valence shells, while the interacting boson–fermion model-3 and 4 (IBFM-3 and IBFM-4) deal with lighter nuclei where protons and neutrons occupy the same valence shell in which case isospin becomes important. These will be discussed in Parts II and III.

1.2 Boson and fermion operators

In the interacting boson–fermion model the collective degrees of freedom are described by boson operators. The properties of these operators were discussed in great detail in Volume 1 and will be only briefly reviewed here. To lowest order of approximation only bosons with angular momentum and parity $J^{\mathrm{P}} = 0^+$ and 2^+ are retained (s and d bosons). The corresponding creation and annihilation operators are written as

$$b^\dagger_{l,m}; \qquad b_{l,m}; \qquad (l = 0, 2; -l \le m \le l), \tag{1.1}$$

or

$$b^\dagger_\alpha; \qquad b_\alpha; \qquad (\alpha = 1, \ldots, 6), \tag{1.2}$$

and satisfy the commutation relations

$$\begin{aligned} [b_{l,m}, b^\dagger_{l',m'}] &= \delta_{ll'}\delta_{mm'}, \\ [b_{l,m}, b_{l',m'}] &= [b^\dagger_{l,m}, b^\dagger_{l',m'}] = 0. \end{aligned} \tag{1.3}$$

or

$$[b_\alpha, b^\dagger_{\alpha'}] = \delta_{\alpha\alpha'}; \qquad [b_\alpha, b_{\alpha'}] = [b^\dagger_\alpha, b^\dagger_{\alpha'}] = 0. \tag{1.4}$$

In addition to collective degrees of freedom, one wants to describe single-particle degrees of freedom. In nuclei, the single particles are protons and neutrons. These are fermions. The angular momentum and parity of these particles depends on the allowed orbits as will be discussed in more detail in Part II. Here we shall denote the angular momentum by j and its z-component by m. An interacting boson–fermion model is specified by the number and the values of angular momenta retained. In treating the single-particle degrees of freedom, it is also convenient to use the formalism of second quantization and introduce the fermion creation and annihilation operators

$$\begin{aligned} &a^{\dagger}_{j,m}, \quad (m = \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \ldots, \pm j), \\ &a_{j,m}, \quad (m = \pm\tfrac{1}{2}, \pm\tfrac{3}{2}, \ldots, \pm j). \end{aligned} \tag{1.5}$$

These operators satisfy anticommutation relations

$$\begin{aligned} &\{a_{j,m}, a^{\dagger}_{j',m'}\} = \delta_{jj'}\delta_{mm'}, \\ &\{a_{j,m}, a_{j',m'}\} = \{a^{\dagger}_{j,m}, a^{\dagger}_{j',m'}\} = 0, \end{aligned} \tag{1.6}$$

where the curly brackets denote an anticommutator, $\{A, B\} = AB + BA$, for any two operators A and B. These have to be contrasted with the commutation relations satisfied by the boson operators, (1.3). There the square brackets denote a commutator, $[A, B] = AB - BA$.

Instead of the double label j, m we shall use, at times, a single index i and denote the operators by

$$a^{\dagger}_{i}; \qquad a_{i}; \qquad (i = 1, \ldots, n), \tag{1.7}$$

with anticommutation relations

$$\{a_i, a^{\dagger}_{i'}\} = \delta_{ii'}; \qquad \{a_i, a_{i'}\} = \{a^{\dagger}_i, a^{\dagger}_{i'}\} = 0. \tag{1.8}$$

Finally, it is assumed that boson and fermion operators commute:

$$[b_{l,m}, a_{j',m'}] = [b_{l,m}, a^{\dagger}_{j',m'}] = [b^{\dagger}_{l,m}, a_{j',m'}] = [b^{\dagger}_{l,m}, a^{\dagger}_{j',m'}] = 0. \tag{1.9}$$

This is a natural assumption if bosons and fermions are elementary particles. In nuclei, where bosons are composite particles (fermion pairs), it is a model assumption. The effects of the compositeness of the bosons are introduced through an additional interaction (exchange interaction).

Spherical tensors can be constructed from the creation and annihilation operators in the usual way. The creation operators already transform in the appropriate way. The annihilation operators do not but one can introduce the operators

$$\tilde{a}_{j,m} = (-)^{j-m} a_{j,-m}, \tag{1.10}$$

that transform appropriately under rotations. With these operators one can form tensor products as discussed in Volume 1. The phase convention (1.10), $(-)^{j-m}$, is chosen to conform with the majority of articles written on the interacting boson–fermion model. This phase is still consistent with that used for the boson operators, $(-)^{l+m}$, Eq. (1.9) of Volume 1, since for bosons (integer l) either choice, $(-)^{l+m}$ or $(-)^{l-m}$, gives the same result.

1.3 Basis states

In the formalism of second quantization, basis states can be constructed by repeated application of creation and annihilation operators on a vacuum state. For bosons the basis is:

$$\mathcal{B}: \qquad b_\alpha^\dagger b_{\alpha'}^\dagger \ldots |\mathrm{o}\rangle, \tag{1.11}$$

while for fermions it is:

$$\mathcal{F}: \qquad a_i^\dagger a_{i'}^\dagger \ldots |\mathrm{o}\rangle. \tag{1.12}$$

Due to their commutation relations, a major difference between boson and fermion operators is that, while one can put any number of bosons in a certain state, one can place only one fermion in the same state. This implies that

$$\left(a_i^\dagger\right)^2 |\mathrm{o}\rangle = 0, \tag{1.13}$$

that is, all indices in (1.12) must be different. The basis of the interacting boson–fermion model is the product of (1.11) and (1.12), usually written as

$$\mathcal{BF}: \qquad a_i^\dagger a_{i'}^\dagger \dots b_\alpha^\dagger b_{\alpha'}^\dagger \dots |\mathrm{o}\rangle. \tag{1.14}$$

Whether fermion operators are written to the left or to the right of boson operators is not relevant since they commute with each other.

It is also here convenient to construct states with good angular momentum by coupling the boson and fermion operators appropriately,

$$\mathcal{BF}: \qquad [[a_j^\dagger \times a_{j'}^\dagger \times \cdots]^{(L_\mathrm{F})} \times [b_l^\dagger \times b_{l'}^\dagger \times \cdots]^{(L_\mathrm{B})}]_M^{(L)} |\mathrm{o}\rangle. \tag{1.15}$$

Since the angular momentum alone is, in general, not sufficient to characterize the states uniquely, one needs extra labels. These will be discussed in Chapter 2.

1.4 Physical operators

1.4.1 The Hamiltonian operator

The model Hamiltonian contains a part that describes the bosons, H_B, a part that describes the fermions, H_F, and a part that describes the interaction between bosons and fermions, V_BF,

$$H = H_\mathrm{B} + H_\mathrm{F} + V_\mathrm{BF}. \tag{1.16}$$

In the interacting boson–fermion model it is assumed that the Hamiltonian conserves separately the number of bosons, N_B, and the number of fermions, N_F. The structure of the various parts of the Hamiltonian operator is then as in Eq. (1.19) of Volume 1,

$$\begin{aligned}
H_\mathrm{B} &= E_0 + \sum_{\alpha\beta} \epsilon_{\alpha\beta} b_\alpha^\dagger b_\beta + \sum_{\alpha\alpha'\beta\beta'} \tfrac{1}{2} u_{\alpha\alpha'\beta\beta'} b_\alpha^\dagger b_{\alpha'}^\dagger b_\beta b_{\beta'} + \cdots, \\
H_\mathrm{F} &= \mathcal{E}_0 + \sum_{ik} \eta_{ik} a_i^\dagger a_k + \sum_{ii'kk'} \tfrac{1}{2} v_{ii'kk'} a_i^\dagger a_{i'}^\dagger a_k a_{k'} + \cdots, \\
V_\mathrm{BF} &= \sum_{\alpha i\beta k} w_{\alpha i\beta k} b_\alpha^\dagger a_i^\dagger b_\beta a_k + \cdots.
\end{aligned} \tag{1.17}$$

This Hamiltonian can be rewritten in such a way that its invariance under rotations becomes evident,

$$
\begin{aligned}
H_{\rm B} = {} & E_0 + \sum_l \epsilon_l \sqrt{2l+1}[b_l^\dagger \times \tilde{b}_l]_0^{(0)} \\
& + \sum_{L_{\rm B}, ll'l''l'''} \tfrac{1}{2} u_{ll'l''l'''}^{(L_{\rm B})} [[b_l^\dagger \times b_{l'}^\dagger]^{(L_{\rm B})} \times [\tilde{b}_{l''} \times \tilde{b}_{l'''}]^{(L_{\rm B})}]_0^{(0)} + \cdots, \\
H_{\rm F} = {} & \mathcal{E}_0 - \sum_j \eta_j \sqrt{2j+1}[a_j^\dagger \times \tilde{a}_j]_0^{(0)} \\
& + \sum_{L_{\rm F}, jj'j''j'''} \tfrac{1}{2} v_{jj'j''j'''}^{(L_{\rm F})} [[a_j^\dagger \times a_{j'}^\dagger]^{(L_{\rm F})} \\
& \times [\tilde{a}_{j''} \times \tilde{a}_{j'''}]^{(L_{\rm F})}]_0^{(0)} + \cdots, \\
V_{\rm BF} = {} & - \sum_{J, ljl'j'} w_{ljl'j'}^{(J)} \sqrt{2J+1} [[b_l^\dagger \times a_j^\dagger]^{(J)} \\
& \times [\tilde{b}_{l'} \times \tilde{a}_{j'}]^{(J)}]_0^{(0)} + \cdots .
\end{aligned}
\tag{1.18}
$$

The coefficients $w_{ljl'j'}^{(J)}$ in (1.18) are the boson–fermion interaction matrix elements,

$$
w_{ljl'j'}^{(J)} = \langle b_l a_j; J | V_{\rm BF} | b_{l'} a_{j'}; J \rangle. \tag{1.19}
$$

Hermiticity of the Hamiltonian imposes further restrictions on the parameters in (1.18). For instance, assuming the matrix elements (1.19) to be real, one finds for the boson–fermion interaction that $w_{ljl'j'}^{(J)} = w_{l'j'lj}^{(J)}$. Other parametrizations of the boson–fermion interaction are possible. Two of them have been frequently used in calculations with the interacting boson–fermion model. They are referred to as the multipole expansion,

$$
V_{\rm BF} = \sum_{L, ll'jj'} w_{ll'jj'}^{\prime(L)} (-)^L \sqrt{2L+1} [[b_l^\dagger \times \tilde{b}_{l'}]^{(L)} \times [a_j^\dagger \times \tilde{a}_{j'}]^{(L)}]_0^{(0)} + \cdots, \tag{1.20}
$$

and the exchange expansion,

$$
V_{\rm BF} = \sum_{J, ljl'j'} w_{ljl'j'}^{\prime\prime(J)} \sqrt{2J+1} : [[b_l^\dagger \times \tilde{a}_j]^{(J)} \times [\tilde{b}_{l'} \times a_{j'}^\dagger]^{(J)}]_0^{(0)} : + \cdots, \tag{1.21}
$$

where the colons $(:\cdots:)$ denote normal ordering. Normal ordering in this case implies that $a^\dagger_{j'}$ should stand on the left of $\tilde{a}_j$ with a minus sign. For these parametrizations, Hermiticity implies the relations $w'^{(L)}_{ll'jj'} = w'^{(L)}_{l'ljj'}$ and $w'^{(L)}_{ll'jj'} = (-)^{j-j'} w'^{(L)}_{ll'j'j}$ in (1.20) and $w''^{(J)}_{ljl'j'} = w''^{(J)}_{l'j'lj}$ in (1.21). The coefficients $w'^{(L)}_{ll'jj'}$ and $w''^{(J)}_{ljl'j'}$ are related to the matrix elements (1.19) in the following way:

$$
\begin{aligned}
w'^{(L)}_{ll'jj'} &= -\sum_J (-)^{j+l'+J}(2J+1)\begin{Bmatrix} l & j & J \\ j' & l' & L \end{Bmatrix} w^{(J)}_{ljl'j'}, \\
w''^{(J)}_{ljl'j'} &= \sum_{J'} (2J'+1)\begin{Bmatrix} l & j' & J' \\ l' & j & J \end{Bmatrix} w^{(J')}_{lj'l'j}.
\end{aligned}
\tag{1.22}
$$

In this expansion, the quantity in curly brackets denotes a Wigner $6j$-symbol (de-Shalit and Talmi, 1963).

In most calculations, only terms up to two creation and two annihilation operators have been retained. In that case, the Hamiltonian H_{B} has been written down explicitly in Volume 1. In order to write down the parts H_{F} and V_{BF} one needs to know the values of j. As an example, we consider the case in which j can take only one value, $j = 3/2$. Omitting the index $j = 3/2$ from the fermion operators, one has

$$
H_{\mathrm{F}} = \mathcal{E}_0 - \eta\sqrt{4}[a^\dagger \times \tilde{a}]^{(0)}_0 + \sum_{L_{\mathrm{F}}=0,2} \tfrac{1}{2} v^{(L_{\mathrm{F}})}[[a^\dagger \times a^\dagger]^{(L_{\mathrm{F}})} \times [\tilde{a}\times\tilde{a}]^{(L_{\mathrm{F}})}]^{(0)}_0,
$$

$$
\begin{aligned}
V_{\mathrm{BF}} = {}& w'^{(0)}_{\mathrm{ss}}[[s^\dagger \times \tilde{s}]^{(0)} \times [a^\dagger \times \tilde{a}]^{(0)}]^{(0)}_0 + w'^{(0)}_{\mathrm{dd}}[[d^\dagger \times \tilde{d}]^{(0)} \times [a^\dagger \times \tilde{a}]^{(0)}]^{(0)}_0 \\
&+ w'^{(1)}_{\mathrm{dd}}[[d^\dagger \times \tilde{d}]^{(1)} \times [a^\dagger \times \tilde{a}]^{(1)}]^{(0)}_0 \\
&+ w'^{(2)}_{\mathrm{dd}}[[d^\dagger \times \tilde{d}]^{(2)} \times [a^\dagger \times \tilde{a}]^{(2)}]^{(0)}_0 \\
&+ w'^{(2)}_{\mathrm{sd}}[[s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} \times [a^\dagger \times \tilde{a}]^{(2)}]^{(0)}_0 \\
&+ w'^{(3)}_{\mathrm{dd}}[[d^\dagger \times \tilde{d}]^{(3)} \times [a^\dagger \times \tilde{a}]^{(3)}]^{(0)}_0.
\end{aligned}
\tag{1.23}
$$

Here we have also used the fact that the Hamiltonian H is an Hermitian operator, $H^\dagger = H$. There are thus three parameters, η, $v^{(0)}$ and $v^{(2)}$, specifying the fermion Hamiltonian H_{F} and six parameters, $w'^{(0)}_{\mathrm{ss}}$, $w'^{(0)}_{\mathrm{dd}}$, $w'^{(1)}_{\mathrm{dd}}$, $w'^{(2)}_{\mathrm{sd}}$, $w'^{(2)}_{\mathrm{dd}}$ and $w'^{(3)}_{\mathrm{dd}}$, specifying the boson–fermion interaction V_{BF} in its multipole form.

1.4.2 Transition operators

Operators inducing electromagnetic transitions of multipolarity L also contain a part describing the bosons, $T_{\rm B}^{(L)}$, and a part describing the fermions, $T_{\rm F}^{(L)}$,

$$T^{(L)} = T_{\rm B}^{(L)} + T_{\rm F}^{(L)}. \tag{1.24}$$

The structure of each term is:

$$\begin{aligned} T_{\rm B}^{(L)} &= t_0^{(0)}\delta_{L0} + \sum_{\alpha\beta} t_{\alpha\beta}^{(L)} b_\alpha^\dagger b_\beta + \cdots, \\ T_{\rm F}^{(L)} &= f_0^{(0)}\delta_{L0} + \sum_{ik} f_{ik}^{(L)} a_i^\dagger a_k + \cdots. \end{aligned} \tag{1.25}$$

In principle, the transition operator $T^{(L)}$ contains also a boson–fermion part, $T_{\rm BF}^{(L)}$, of the form

$$T_{\rm BF}^{(L)} = \sum_{ik\alpha\beta} r_{ik\alpha\beta}^{(L)} a_i^\dagger a_k b_\alpha^\dagger b_\beta + \cdots. \tag{1.26}$$

This part, however, contains at least two creation and two annihilation operators and is usually neglected. Again, since the operators $T^{(L)}$ must transform as tensors of rank L under rotations, it is more convenient to rewrite (1.25) in coupled-tensor form,

$$\begin{aligned} T_{{\rm B},\mu}^{(L)} &= t_0^{(0)}\delta_{L0} + \sum_{ll'} t_{ll'}^{(L)} [b_l^\dagger \times \tilde{b}_{l'}]_\mu^{(L)} + \cdots, \\ T_{{\rm F},\mu}^{(L)} &= f_0^{(0)}\delta_{L0} + \sum_{jj'} f_{jj'}^{(L)} [a_j^\dagger \times \tilde{a}_{j'}]_\mu^{(L)} + \cdots. \end{aligned} \tag{1.27}$$

In addition to being tensors under rotations, the electromagnetic transition operators have a definite character under parity. If only bosons with $J^{\rm P} = 0^+$ and 2^+ are considered, the parity of the boson part $T_{\rm B}^{(L)}$ is always positive. The fermion part $T_{\rm F}^{(L)}$ instead can have either positive or negative parity. As will be discussed in Part II, for each fermion the angular momentum j is built from an orbital angular momentum l_j and a spin $s = 1/2$. Its parity is thus $(-)^{l_j}$. When combining fermion creation, $a_j^\dagger$, and annihilation, $\tilde{a}_{j'}$, operators one must make sure that the combined parity is equal to that of the transition. This implies that the coefficients $f^{(L)}_{jj'}$ vanish unless $(-)^{l_j+l'_j+L}$ is $+1$ for electric transitions and -1 for magnetic transitions.

Usually, only terms containing one creation and one annihilation operator are retained. The explicit form of $T^{(L)}_{\mathrm{B}}$ is then given in Eq. (1.24) of Volume 1. In order to write down $T^{(L)}_{\mathrm{F}}$ one needs to know the values of j. As an example, we consider again the case $j = 3/2$ for which one obtains operators with multipolarity $L = 0, 1, 2, 3$,

$$\begin{aligned} T^{(\mathrm{E0})}_{\mathrm{F},0} &= f^{(0)} + f'^{(0)}[a^\dagger \times \tilde{a}]^{(0)}_0 , \\ T^{(\mathrm{M1})}_{\mathrm{F},\mu} &= f^{(1)}[a^\dagger \times \tilde{a}]^{(1)}_\mu , \\ T^{(\mathrm{E2})}_{\mathrm{F},\mu} &= f^{(2)}[a^\dagger \times \tilde{a}]^{(2)}_\mu , \\ T^{(\mathrm{M3})}_{\mathrm{F},\mu} &= f^{(3)}[a^\dagger \times \tilde{a}]^{(3)}_\mu . \end{aligned} \tag{1.28}$$

1.4.3 Independent parameters

Some of the parameters in the Hamiltonian and the transition operators can be eliminated, by using the condition that the number of bosons, N_{B}, and fermions, N_{F}, is conserved. For example, one of the terms in V_{BF}, (1.23), can be eliminated to yield

$$\begin{aligned} H_{\mathrm{F}} &= \mathcal{E}_0 - \eta' \sqrt{4} [a^\dagger \times \tilde{a}]^{(0)}_0 \\ &\quad + \sum_{L_{\mathrm{F}}=0,2} \tfrac{1}{2} v^{(L_{\mathrm{F}})} [[a^\dagger \times a^\dagger]^{(L_{\mathrm{F}})} \times [\tilde{a} \times \tilde{a}]^{(L_{\mathrm{F}})}]^{(0)}_0 \end{aligned}$$

$$\begin{aligned} V_{\mathrm{BF}} &= w''^{(0)}_{dd} [[d^\dagger \times \tilde{d}]^{(0)} \times [a^\dagger \times \tilde{a}]^{(0)}]^{(0)}_0 + w'^{(1)}_{dd} [[d^\dagger \times \tilde{d}]^{(1)} \times [a^\dagger \times \tilde{a}]^{(1)}]^{(0)}_0 \\ &\quad + w'^{(2)}_{dd} [[d^\dagger \times \tilde{d}]^{(2)} \times [a^\dagger \times \tilde{a}]^{(2)}]^{(0)}_0 \\ &\quad + w'^{(2)}_{sd} [[s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}]^{(2)} \times [a^\dagger \times \tilde{a}]^{(2)}]^{(0)}_0 \\ &\quad + w'^{(3)}_{dd} [[d^\dagger \times \tilde{d}]^{(3)} \times [a^\dagger \times \tilde{a}]^{(3)}]^{(0)}_0 , \end{aligned} \tag{1.29}$$

with

$$\eta' = \eta - \frac{N_{\mathrm{B}}}{\sqrt{4}} w'^{(0)}_{\mathrm{ss}} , \qquad w''^{(0)}_{\mathrm{dd}} = w'^{(0)}_{\mathrm{dd}} - \sqrt{5} w'^{(0)}_{\mathrm{ss}} . \tag{1.30}$$

This elimination reduces the number of parameters in V_{BF} by one, and is similar to that discussed in Volume 1.

1.4.4 Special forms of the boson–fermion interaction

The form (1.18) of V_{BF} is too general for a purely phenomenological analysis. On the basis of microscopic considerations, to be discussed in Volume 3, it has been suggested (Iachello and Scholten, 1979) that a simpler form may account for most of the observed properties. In this simpler form only three terms are retained. First, a monopole term, written in the form

$$V_{\mathrm{BF}}^{\mathrm{MON}} = \sum_j A_j [[d^\dagger \times \tilde{d}]^{(0)} \times [a_j^\dagger \times \tilde{a}_j]^{(0)}]_0^{(0)}; \tag{1.31}$$

second, a quadrupole term, written in the form

$$V_{\mathrm{BF}}^{\mathrm{QUAD}} = \sum_{jj'} \Gamma_{jj'} [[(s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}) + \chi(d^\dagger \times \tilde{d})]^{(2)} \times [a_j^\dagger \times \tilde{a}_{j'}]^{(2)}]_0^{(0)}; \tag{1.32}$$

and finally, an interaction, called exchange interaction, that takes into account the fact that the bosons are fermion pairs (Talmi, 1981; Scholten and Dieperink, 1981; Otsuka *et al.*, 1987). This interaction can be written as

$$V_{\mathrm{BF}}^{\mathrm{EXC}} = \sum_{jj'j''} \Lambda_{jj'}^{j''} : [[d^\dagger \times \tilde{a}_j]^{(j'')} \times [\tilde{d} \times a_{j'}^\dagger]^{(j'')}]_0^{(0)} :, \tag{1.33}$$

and has been shown (Yoshida *et al.*, 1988) to be of crucial importance in reproducing the signature dependence of electromagnetic transitions in odd–even nuclei. Thus, two terms are taken from the multipole expansion (1.20) and one term from the exchange expansion (1.21). They can be converted to two-body matrix elements (1.19) using the relations (1.22). In the exchange interaction one could also have the terms

$$V_{\mathrm{BF}}^{\mathrm{EXC}'} = \sum_{jj'} \Lambda_{jj'}^{\prime j'} : \{[[d^\dagger \times \tilde{a}_j]^{(j')} \times [\tilde{s} \times a_{j'}^\dagger]^{(j')}]_0^{(0)} + [[s^\dagger \times \tilde{a}_{j'}]^{(j')} \times [\tilde{d} \times a_j^\dagger]^{(j')}]_0^{(0)}\} :, \tag{1.34}$$

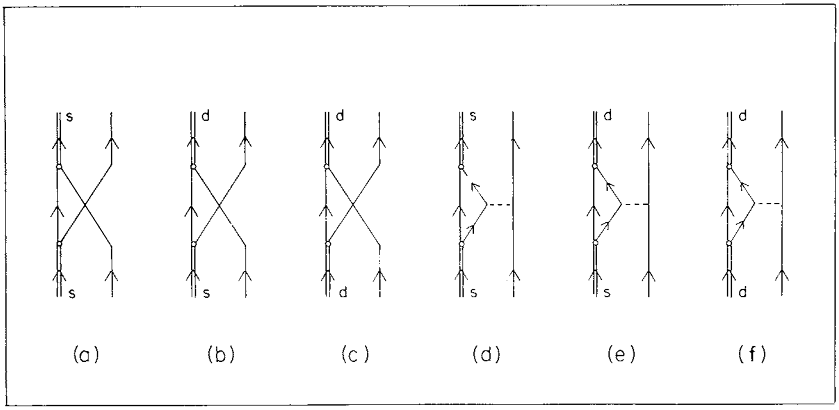

Fig. 1-1 Schematic representation of the boson–fermion interaction. Graphs (a), (b) and (c) represent the exchange interaction, while graphs (d), (e) and (f) represent the direct interaction.

and

$$V_{\rm BF}^{\rm EXC''} = \sum_{j} \Lambda_{jj}^{''j} : [[s^\dagger \times \tilde{a}_j]^{(j)} \times [\tilde{s} \times a_j^\dagger]^{(j)}]_0^{(0)} : . \qquad (1.35)$$

If only one single-particle orbit j is taken into account, these two terms can be eliminated and included in (1.32) and (1.33), respectively. When several j orbits are taken into account, these terms must be retained since their dependence on the indices j and j' is different from that of (1.32) and (1.33). Microscopic calculations also indicate the presence of a direct interaction of the type (1.19) between a fermion and a d boson (Talmi, 1981; Gelberg, 1983).

The boson–fermion interaction can be displayed graphically. This is usually done by denoting the bosons by a double line, since they are fermion pairs, the fermion by a single line and the interaction by a wavy line. The direct and exchange interactions are then displayed as in Fig. 1.1.

1.4.5 Transfer operators

Another set of operators of particular interest in nuclear physics is formed by transfer operators. In the interacting boson–fermion model, two types of transfer are possible, transfer of a nucleon pair and transfer of a single nucleon. Two-nucleon transfer operators are written entirely in terms of boson operators and were discussed in Volume 1. The form of the operators describing a one-nucleon transfer reaction depends on whether the number of bosons is conserved or is changed by one. In the first case, the operators are, in lowest approximation, given by the corresponding creation and annihilation operators, schematically written as

$$\begin{aligned} P_{+}^{(j)} &= \sum_i p_i^{(j)} a_i^\dagger, \\ P_{-}^{(j)} &= \sum_i p_i^{(j)} a_i, \end{aligned} \tag{1.36}$$

or, more explicitly, as

$$\begin{aligned} P_{+,m}^{(j)} &= p_j a_{j,m}^\dagger, \\ P_{-,m}^{(j)} &= p_j \tilde{a}_{j,m}, \end{aligned} \tag{1.37}$$

where j denotes the transferred angular momentum. However, for the transfer operators (1.37), it has been found that the lowest order is not sufficient to describe the experimental situation, since it does not take into account the composite nature of the bosons. Consequently, one needs to introduce higher-order terms. To next order, these are written as

$$\begin{aligned} P_{3,+}^{(j)} &= \sum_{\alpha\beta i} q_{\alpha\beta i}^{(j)} b_\alpha^\dagger b_\beta a_i^\dagger, \\ P_{3,-}^{(j)} &= \sum_{\alpha\beta i} q_{\alpha\beta i}^{(j)} b_\beta^\dagger b_\alpha a_i, \end{aligned} \tag{1.38}$$

or, in coupled-tensor form,

$$\begin{aligned} P_{3,+,m}^{(j)} &= \sum_{ll',k,j'} q_{ll',k,j'}^{(j)} [[b_l^\dagger \times \tilde{b}_{l'}]^{(k)} \times a_{j'}^\dagger]_m^{(j)}, \\ P_{3,-,m}^{(j)} &= \sum_{ll',k,j'} (-)^{l+l'-k} q_{ll',k,j'}^{(j)} [[b_{l'}^\dagger \times \tilde{b}_l]^{(k)} \times \tilde{a}_{j'}]_m^{(j)}, \end{aligned} \tag{1.39}$$

where we have explicitly introduced the phase $(-)^{l+l'-k}$ for the operators to transform as spherical tensors under rotations. In the second case, when the number of bosons is changed by one, the transfer operators are in lowest order

$$
\begin{aligned}
P_+^{\prime(j)} &= \sum_{\alpha i} p_{\alpha i}^{\prime(j)} b_\alpha a_i^\dagger, \\
P_-^{\prime(j)} &= \sum_{\alpha i} p_{\alpha i}^{\prime(j)} b_\alpha^\dagger a_i,
\end{aligned} \tag{1.40}
$$

or, more explicitly,

$$
\begin{aligned}
P_{+,m}^{\prime(j)} &= \sum_{lj'} p_{lj'}^{\prime(j)} [\tilde{b}_l \times a_{j'}^\dagger]_m^{(j)}, \\
P_{-,m}^{\prime(j)} &= \sum_{lj'} p_{lj'}^{\prime(j)} [b_l^\dagger \times \tilde{a}_{j'}]_m^{(j)}.
\end{aligned} \tag{1.41}
$$

The operators with index + describe a transfer reaction from an even–even to an even–odd nucleus, while the operators with index − describe the inverse reaction.

1.4.6 Special forms of the transfer operators

It has been suggested (Scholten, 1980) that a special form of the higher-order terms in the transfer operators is often sufficient to describe the experimental situation. In this form only two terms are retained in (1.39). The first is a monopole term,

$$
P_{3,+,m}^{\mathrm{MON}(j)} = q_0^{(j)} [[d^\dagger \times \tilde{d}]^{(0)} \times a_j^\dagger]_m^{(j)}. \tag{1.42}
$$

The second is a quadrupole term,

$$
P_{3,+,m}^{\mathrm{QUAD}(j)} = \sum_{j'} q_{2,j'}^{(j)} [[(s^\dagger \times \tilde{d} + d^\dagger \times \tilde{s}) + \chi (d^\dagger \times \tilde{d})]^{(2)} \times a_{j'}^\dagger]_m^{(j)}. \tag{1.43}
$$

Similar expressions hold for the substraction operators $P_{3,-,m}^{(j)}$. More realistic but more complicated transfer operators have recently been proposed by Sofia and Vitturi (1989) on the basis of microscopic theory.